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# Stability of D1-strings inside a D3-brane

**Louis Leblond and S.-H. Henry Tye**

*Laboratory for Elementary-Particle Physics, Cornell University*
*Ithaca, NY 14853, U.S.A.*
*E-mail:* lleblond@lepp.cornell.edu, tye@lepp.cornell.edu

Abstract: Within the tachyon condensation approach, we find that a D$(p-2)$-brane is stable inside D$p$-branes when the bulk is compactified. It is a codimension-2 soliton of the D$p$-brane action with coupling to the bulk $(p-1)$-form RR field. We discuss the properties of such solitons. They may appear as detectable cosmic strings in our universe.

Keywords: Superstrings and Heterotic Strings, D-branes, Tachyon Condensation, Cosmology of Theories beyond the SM.



 

## Contents



## 1. Introduction

The evidence that the early universe has gone through an inflationary epoch has become very strong. In the brane world scenario, where the standard model fields (i.e., photons, electrons, quarks, etc., except graviton) are open string modes on branes, brane inflation [1] is quite natural. A particularly simple version of brane inflation involves the slow motion of a D3-brane towards an anti-D3-brane [2]–[6]. Recently, based on a realistic superstring compactification where all moduli of the vacuum are stabilized [7, 8], it is shown that this D3-brane pair inflationary scenario may be realized in superstring theory [9]–[12]. Inflation ends as the D3-brane pair annihilates. Suppose the standard model fields live in a stack of (anti-)D3-branes. To allow the energy released to heat up the universe to start the hot radiation dominated big bang era [13, 14], the D3-brane pair annihilation should happen close to this stack of branes.



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Towards the end of the inflationary epoch, cosmic strings are produced [5, 15, 16, 17, 18, 19, 20, 21]. In the above scenario, the annihilation of the D3-brane pair produces cosmic strings that are D1-strings (i.e., D1-branes) [22]. The tension of such D1-strings are estimated to be around $G\mu \simeq 10^{-9}$ to $10^{-10}$ [15, 17]. If they survive long enough to evolve as a cosmic string network [23], they will produce distinct signatures that should be detected in the near future, in particular by gravitational wave detectors such as LIGO II/Virgo or LISA [24].

If the D3-brane collides with a stack of anti-D3-branes and annihilates one of them, D1-strings are produced inside (or very close to) the (anti-)D3-branes [22]. However, in string theory, D1-strings and D3-branes are not BPS with respect to each other. In fact, it is generally believed that the D1-strings will dissolve inside a D3-brane [25, 26]; that is, their energy will spread throughout the D3-brane. In this case, the cosmic strings would have dissolved almost immediately after they were formed and no observable signature will be left. So the (in)stability of such D1-strings inside a D3-brane is a very important phenomenological issue. Of course, this question in string theory is interesting in its own right.

The possibility of the stability of D1-strings inside a D3-brane was recently pointed out by Copeland, Myers and Polchinski (CMP) [17]. The coupling of the RR 2-form field $C_2$ to the abelian gauge field $A_1$ inside the D3-brane is finite when the extra 6 dimensions are compactifed. This leads to spontaneous symmetry breaking and a D1-string inside a D3-brane becomes a topologically stable vortex (a D1-vortex) with localized energy (tension) density inside a D3-brane. However, this vortex is no longer BPS and has a net zero RR charge (but non-zero charge density) as measured by $C_2$ inside the D3-brane, since the winding number contribution to this charge is canceled by the magnetic flux contribution. Because of the conservation of the winding number, this same vortex becomes a D1-string when moved outside a D3-brane, as expected.

To justify the action used here and in CMP, we present a topological argument on the stability of this D1-vortex in the context of tachyon condensation, where a D1-string also appears as a vortex. Although our approach uses boundary superstring field theory, the D1-vortex stability is based on topological reasonings and so is insensitive to the details of the particular framework used. This approach also reveals the relation between a D1-string outside a D3-brane (a BPS vortex due to tachyon condensation) and a D1-vortex inside a D3-brane (a vortex due to the $C_2$ coupling). Although the dynamics is somewhat involved, this transition as a D1-string moves in/out a D3-brane is expected to be smooth. In the limit of vanishing $C_2$ coupling (e.g., some of the extra dimensions decompactfy), the size of such a D1-vortex grows to infinity. In effect, the magnetic flux spreads and a D1-string dissolves inside a D3-brane. It is the $C_2$ coupling that stabilizes the D1-string inside a D3-brane.

More generally, a D$(p-2)$-brane is stable inside a stack of D$p$-branes. In a brane world where $(p-3)$ dimensions of the D$p$-branes are compactified while the remaining 3 dimensions span the universe, D$p$-anti-D$p$-brane inflation would generically create stable cosmic strings that are D$(p-2)$-brane defects (with $(p-3)$ dimensions compactified). Their individual stability allows the cosmic string network evolution and implies that the detection of signatures of cosmic strings should be a good test of the brane inflationary scenario; furthermore, it gives an eagerly sought window to the superstring theory itself. Of course, the details of the phenomenology may be quite sensitive to the specific inflationary scenario.

The organization of this paper is as follows. In section 2, we review the old argument why a D1-brane (i.e., a D1-string) was believed to dissolve inside a D3-brane. We also review the argument why there should be a domain wall enclosed by a D1-string. These two features constitute something of a puzzle, as pointed out in CMP. We then summarize our resolution to this puzzle. In section 3, we solve analytically the vortex solution of the model involving the gauge field $A_1$ and $C_2$ inside a D3-brane. The $C_2$ coupling to $A_1$ leads to the Green-Schwarz mechanism where the gauge field becomes massive. We show that this model admits a topologically stable vortex solution, which is identified as a D1-vortex, that is, a D1-string inside a D3-brane, even though this vortex is not BPS. We show it has localized energy density, not spread throughout the D3-brane. However, the tension of this vortex is logarithmically divergent. In section 4, we give the physical picture. By comparing to the abelian Higgs model, we argue that the divergence in tension is expected in the approximate nature of the model, and is easily cured by adding the massive mode (namely the radial Higgs field associated to the axion) to the low energy effective supergravity action. In section 5, we study this problem in the framework of tachyon condensation, where we give a topological argument why the D1-vortex is stable inside a D3-brane. We also use the boundary superstring field theory models to draw the connection between the two approaches.

## 2. The CMP puzzle

In the cosmological context, we want to know if a very large D1-string loop can survive over cosmological time scale. If so, it would interact with other D1-strings and evolve as a component in a cosmic string network; it would also oscillate and radiate gravitational waves which may be detected. On the other hand, if it either dissolves in superstring/Planck time scale, or shrinks rapidly due to the presence of a domain wall, then there is no observable signal left for us to detect today. Let us briefly review the status of a D1-string as known in the literature. We explain the CMP puzzle [17] and then summarize our resolution.

### 2.1 Basic idea of dissolution

Consider a D1-string that is either inside a D3-brane, or outside a D3-brane. If it is outside, closed string exchange between them is attractive, so they will quickly come into contact. In either case, we are led to study a D1-D3 system. It is well known that the D$p$+D$(p-2)$ system is a non-BPS state with a tachyon in its spectra. This state is unstable and there exists a BPS bound state with the same RR charge but lower mass:

$$M_{Dp+D(p-2)} = \tau_p V_p + \tau_{p-2} V_{p-2} \geq (\tau_p^2 V_p^2 + \tau_{p-2}^2 V_{p-2}^2)^{\frac{1}{2}}\,, \tag{2.1}$$

where $\tau_p$ is the tension of a D$p$-brane. It is argued [27, 28, 29] that a D$p$ brane with a constant flux throughout its volume has the correct tension and charge to be interpreted as the BPS bound state. It is then logical to interpret the decay of the D$p$ + D$(p-2)$ system to its BPS bound state as the dissolution of the D$(p-2)$-brane into a constant flux on the D$p$-brane, that is, the D$(p-2)$-brane is 'smeared' out throughout the D$p$-brane [25, 26]. This dissolution means a D$(p-2)$-brane is unstable inside a D$p$-brane. In the brane world scenario where the D3-branes span our universe, the D1-string would have dissolved immediately after their production. If the D1-string intersects with a D3-brane along some curve inside the D3-brane, the D1-string is expected to break with flux lines connecting the two ends. The spreading of these flux lines and the energy density will signal the breaking of the D1-string by the presence of the D3-brane.

### 2.2 Axionic string and domain wall

The presence of a domain wall bounded by a string was first pointed out by Witten [30]. In four large spacetime dimensions, a D1-string is charged under the 2-form RR field $C_2$, which is dual to an axion $\phi$: $\partial_\mu \phi \simeq \epsilon_{\mu\nu\rho\sigma}\partial^\nu C^{\rho\sigma}$. That is, $\phi(x)$ increases by $2\pi$ as it circles the D1-string once. In this sense, D1-strings are axionic strings. Since we have not seen a massless axion, we expect the Peccei-Quinn $\mathrm{U}(1)_{PQ}$ to be broken (say to $Z_k$, $k \geq 1$) and the axion picks up a mass. Then the winding of $\phi$ implies a domain wall with the D1-string as its boundary. To get some idea of the domain wall tension, assume a potential $M^4(1-\cos\phi)$ that generates an axion mass $m_\phi$ and gives a domain wall tension around $M^4/m_\phi$. Putting in an allowed value for the axion mass ($m_\phi \sim 10^{-14}\,\mathrm{GeV}$) and assume $M$ to be of the order of the superstring scale, we get a very large domain wall tension. For any reasonable values of $m_\phi$ and $M$ (even say $M \sim m_\phi$), the domain walls confine the D1-strings. That is, the D1-string loops will rapidly shrink and disappear, leaving no lasting cosmological signatures to be detected.

### 2.3 The CMP puzzle and its resolution

As explained above, there are two ways to get rid of the D1-strings as cosmic strings in the early universe. However, these two ways are incompatible with each other. This is the puzzle pointed out by CMP; since the boundary of a boundary is zero, the presence of a domain wall implies that a D1-string cannot break. On the other hand, if it breaks, i.e., the D1-strings develop ends, then the domain wall cannot exist.

Here, we present the resolution to this puzzle. We find that, for finite $C_2$ coupling to the abelian field strength, the D1-strings do not break. In the tachyon condensation approach, where even a D1-string outside a D3-brane is treated as a vortex, one simply look at the topology/structure of the vacuum degeneracy and argue that a D1-vortex is stable both inside and outside a D3-brane. We see that a D1-string outside a D3-brane is a BPS vortex due to tachyon condensation while a D1-vortex inside a D3-brane is a vortex due to the $C_2$ coupling. However, the actual dynamics of the transition between a D1-string and a D1-vortex as it moves in/out of a D3-brane is expected to be smooth.

Once its stability is established, one can then write a simple effective action to study the properties of such a vortex. Inside a D3-brane, the axion becomes the would-be Goldstone mode that is swallowed by the gauge field inside the D3-brane via the Higgs (or Green-Schwarz) mechanism. Since the axion remains massless, the domain wall tension is exactly zero, i.e., there is no domain wall. At the same time, the D1-string becomes a vortex (a D1-vortex) in the abelian Higgs model, with localized energy density (see figure 2), though it is no longer BPS. As a consequence, the D1-strings survive inside D3-branes and they will evolve as a network of cosmic strings in our universe. The actual phenomenology of the cosmic string network does depend on the details of the inflationary scenario.

As measured by $C_2$, the net RR charge of this D1-vortex is zero. Besides a positive contribution to the RR charge from the winding number of the axion, there is a negative contribution to the RR charge coming from the magnetic flux. This is screening. (However, as shown in figure 1, these two contributions do not cancel locally, so there is a non-trivial RR charge density.) That is, as we move a D1-string inside a D3-brane, it loses its RR charge, but retains its winding number. (One can define this same charge in the abelian Higgs model, and likewise, a vortex there also has a net zero charge.) Note that the winding number is identified with the RR charge outside the D3-brane. Since it is conserved when a D1-string moves inside the D3-brane (it becomes the winding number of the D1-vortex), it is a more useful quantum number to keep track.

## 3. The stability of a D1-string inside a D3-brane

To set up the problem, we first consider a D1-string outside a D3-brane, i.e., a D1-string as a BPS D1-brane with coupling to the 2-form RR field $C_2$. Next we put it inside a D3-brane. The key new ingredient is the coupling of the abelian gauge field $A_1$ to $C_2$. The Green-Schwarz mechanism takes place and $A_1$ becomes massive. This model can be exactly solved where the D1-string becomes a vortex in the abelian Higgs model. This D1-vortex remains topologically stable with localized energy density, though they are no longer BPS. There is a very rich literature [31] on Chern-Simons vortices and relation between Chern-Simons terms and the abelian Higgs model, though in different contexts.

### 3.1 D1-string outside a D3-brane

Let us dimensionally reduce the 10-dimensional theory to an effective 4-dimensional theory to get the kinetic term for C$_2$ from the bulk action. Consider $n$ D1-strings coupled to $C_2$ in the four uncompactified dimensions, all sitting along the $z$-axis at $r = |x_\perp| = \sqrt{x^2+y^2} = 0$. After rescaling $C_2$ to obtain a canonical kinetic term, we have (with constant dilaton background):

$$S = -\int_{M^4} \frac{1}{2}|dC_2|^2 + 2\pi n a \delta^2(x_\perp) \wedge C_2 \,, \tag{3.1}$$

where $a = \sqrt{2}\tau_1\kappa_4$ measures the RR charge of a D1-string whose coordinate is $\delta^2(x_\perp)dx\wedge dy$. $\tau_1$ is the D1-string tension and $\kappa_4$ is the 4-dimensional effective gravitational coupling. It is related to the 10-dimensional coupling $\kappa_4^2 V_6 = \kappa^2 = \kappa_{10}^2 g_s^2$ where $V_6$ is the 6-dimensional compactified volume and $g_s$ is the string coupling. Introducing the dual of $C_2$, $ad\phi = \star dC_2$, we have

$$\begin{aligned} a d\wedge d\phi &= d\star dC_2 = 2\pi n a\delta^2(x_\perp)\,, \\ \star dC_2 &= ad\phi = a\frac{n}{r}\,, \end{aligned} \tag{3.2}$$

where we have chosen $\phi$ to be dimensionless and $\star = \star_4$ unless we specify otherwise. Note that $d \wedge d\phi$ is not identically zero since $\phi$ is not single valued; $\phi(x_\mu)$ increases by $2\pi n$ as it circles the $z$-axis once.

### 3.2 D1-string inside a D3-brane

Now consider the same D1-string inside a D3-brane, with the D3-brane world volume action that involves the $C_2$ and the abelian gauge field $A_1$.

$$S = -\int_{M_4} \frac{1}{2}|G_2|^2 + \frac{1}{2}|dC_2|^2 + \xi C_2 \wedge G_2 + 2\pi n a \delta^2(x_\perp) \wedge C_2 \tag{3.3}$$

where $G_2 = dA_1$, $\xi = \sqrt{2\tau_3}\kappa_4$, $\tau_3$ is the D3-brane tension. Note that $C_2$, $A_1$, $\xi$ and $a$ have dimension of mass. For $\xi = 0$ (e.g., when $V_6 \to \infty$), both $A_1$ and $C_2$ are massless. For finite $\xi$, spontaneous symmetry breaking via the Green-Schwarz mechanism takes place.

The equations of motion for this action are:

$$d \star dA_1 = \xi dC_2\,, \tag{3.4}$$

$$d \star dC_2 = \xi G_2 + 2\pi n a \delta^2(x_\perp)\,. \tag{3.5}$$

There are two ways to solve these equations. First let us solve for $C_2$. The solution of (3.5) is (in non-compact space):

$$\star dC_2 = \xi A_1 + a d\phi\,, \tag{3.6}$$

where, by analogy with the solution outside the D3-brane, we get the $\delta$ function from the multi-valued $\phi$. Putting this back in eq. (3.4) we get:

$$d \star dA_1 = \star(a\xi d\phi + \xi^2 A_1)\,. \tag{3.7}$$

For the ground state (i.e., $n = 0$), where $\phi = 0$ (or is a pure gauge), we see that $A_1$ has mass $\xi$. The other way to solve this set of coupled equations (3.4), (3.5) is to first solve for $A_1$,

$$\star dA_1 = \xi C_2 + d\eta_1 \to \xi C_2 \tag{3.8}$$

since we are considering vortices without electric flux, so we expect $d\eta_1 = 0$. Putting this back into the equation for $C_2$, we have

$$d \star dC_2 = (-) \star \xi^2 C_2 + 2\pi n a \delta^2(x_\perp) \tag{3.9}$$

where $C_2$ has mass $\xi$. We can view this system as

- the gauge field $A_1$ swallowing an axion (the Goldstone boson) $\phi$ (where $d\phi = \star dC_2 + \cdots$) becoming massive: $A_1 \to A_1 + a d\phi/\xi$;

or

- the 2-form field $C_2$ swallowing (the dual of) the gauge field $A_1$ and becoming massive (where $d\eta_1 = \star dA_1 + \cdots$): $C_2 \to C_2 + d\eta_1/\xi$.

Classically, these two dual pictures describe the same physics, as a massive $C_2$ or a massive $A_1$ has three physical degrees of freedom in four dimensions.

A comment on the validity of the above effective action (3.3) is in order. It is well-known that there is an open string tachyonic mode stretching between a D1-string and a D3-brane when they are close to each other. It is precisely the presence of such a tachyonic mode that signals the instability of the D1-D3 system. This may suggest that, in the study of the stability of a vortex solution, we should include such a mode in the effective action. However, it is not clear whether such a tachyonic mode is present when we include the induced magnetic flux on the D3-brane, i.e., the induced flux may have removed (lifted the mass squared of) the tachyon field. To see that there is no tachyonic mode to destabilize the solution, we shall turn to tachyon condensation in section 5. By including all relevant tachyonic modes in the beginning, we examine how spontaneous symmetry breaking take place and keep track of the tachyon modes and possible solitons. The topology of the degeneracy of the final vacuum dictates the stability of any possible defect. In this approach, a D1-string appears as a BPS vortex and we can follow it as it moves inside a D3-brane. Although tachyon condensation is analyzed in the boundary superstring field theory framework, the stability argument is based on topology and so should be independent on any of the details. There we find that the D1-vortex is topologically stable, justifying the above action.

### 3.3 Vortex solutions inside a D3-brane

Here we are interested in any possible vortex solution of this model. As we shall see, the $A_1$ description (3.7) is exactly a limiting case of the abelian Higgs model and the physics is completely transparent, while the $C_2$ picture (3.9) is less familiar. Fortunately, the vortex solution of this model has an explicit analytic form, so it is relatively easy to clarify the relation between these two pictures. There is a more compelling reason that we prefer to consider the $A_1$ description (3.7) as more fundamental. This equation allows multi-vortices at different locations and orientations as its solitonic solutions. In the $C_2$ picture, these vortices are sourced by $\delta$-functions input by hand.

Let us present the solution in details. The equation of motion (3.7) for the gauge field translates to

$$\partial_\mu G^{\mu\nu} = (a\xi\partial^\nu\phi + \xi^2 A^\nu) = J^\nu \,. \tag{3.10}$$

The solution we are interested in is a vortex with winding number $n$. Since the winding number is topological and should be independent of detailed dynamics, we expect $\phi(x)$ to increase by $2\pi n$ as it circles the $z$-axis once. We choose to work in the Coulomb gauge $\partial_\mu A^\mu = 0$. So we start with the following ansatz, in cylindrical coordinates $(t, z, r, \theta)$:

$$\begin{aligned} \phi(x) &= n\theta\,, \\ A_\theta(x) &= -\frac{an}{\xi r}\alpha(r) \end{aligned} \tag{3.11}$$

where $n$ is the winding number of the vortex. This leads to

$$\frac{d^2\alpha}{dr^2} - \frac{1}{r}\frac{d\alpha}{dr} - \xi^2(\alpha - 1) = 0\,, \tag{3.12}$$

which reduces to the modified Bessel equation. Let $u = \xi r$, the solution is just

$$\alpha(u) = 1 - uK_1(u)\,, \tag{3.13}$$

where $K_1(u)$ is the modified Bessel function of order 1. This is valid for all $r \geq 0$. For small $u$:

$$\begin{aligned} K_1(u) &\sim \frac{1}{u} + \frac{u}{2}\ln\left(\frac{u}{2}\right), \\ K_0(u) &\sim -\ln\left(\frac{u}{2}\right) - \gamma\,, \end{aligned} \tag{3.14}$$

and for large $u$,

$$K_1(u) \simeq K_0(u) \simeq \sqrt{\frac{\pi}{2u}}e^{-u}\,, \tag{3.15}$$

where we have given the properties of $K_0(u)$ as well. The following relations are useful (for $u > 0$):

$$\begin{aligned} \frac{d}{du}(K_0) &= -K_1\,, \\ \frac{d}{du}(uK_1) &= -uK_0\,. \end{aligned} \tag{3.16}$$

Using eq. (3.11), (3.13), we see that the magnetic field $\mathbf{B}$ is along the $\hat{z}$-direction,

$$\begin{aligned} B_z &= \frac{1}{r}\partial_r(rA_\theta) \\ &= -\frac{2\pi na}{\xi}\delta^2(x_\perp) - an\xi K_0(u) + \frac{2\pi na}{\xi}\delta^2(x_\perp) \\ &= -an\xi K_0(u)\,, \end{aligned} \tag{3.17}$$

where the first $\delta$-function comes from the $r^{-1}$ term in $A_\theta$ while the second $\delta$-function together with the $K_0$ term comes from the $K_1$ term in $A_\theta$. It is easy to check this using Stokes' law for each term in $A_\theta$. Integrating over the $xy$-plane, we obtain the total magnetic flux of the vortex,

$$\int B_z r dr d\theta = -2\pi n\frac{a}{\xi}\,. \tag{3.18}$$

As we shall see, $\xi/a = \sqrt{\tau_3}/\tau_1$ corresponds to the electric charge in the abelian Higgs model. Now we can describe this same vortex solution in terms of $C_2$ and $d\eta_1$. With the above vortex, only the $(tz)$ component of $C_2$ is non-trivial. Using eq. (3.8) (or integrating eq. (3.6)), we have

$$C_{01} = -anK_0(u)\,. \tag{3.19}$$

Note that $d\eta_1 = 0$, as expected, since it is the dual of $B_z$, i.e., the electric field, which is absent here. Alternatively, we can solve for $C_2$ from eq. (3.9). With only the $(tz)$ component of $C_2$, we have

$$\partial_\perp^2 C_{01} = \frac{1}{r}\partial_r(r\partial_r C_{01}) = \xi^2 C_{01} + 2\pi na\delta^2(x_\perp) \tag{3.20}$$



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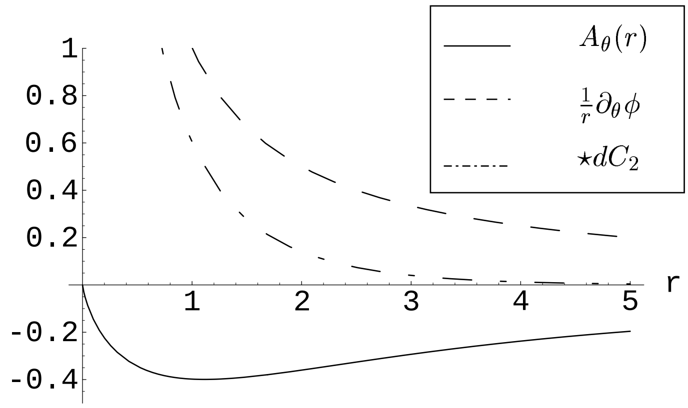


**Figure 1:** $A_\theta$ (solid line), $d\phi$ (dashed line) and $\star dC_2$ (dot-dashed line), where we have set $a = \xi = n = 1$. $d\phi$ is canceled by $A_1$ to screen the RR charge, so $\star dC_2$ drops exponentially for large $r$.

which is precisely the modified Bessel equation for $K_0$ once we account for the $\delta$-function correctly (similar to the second $\delta$-function in eq. (3.17)). So we recover the solution given in eq. (3.19). Note that, for large $r$, $C_{01} \sim e^{-\xi r}/\sqrt{r}$. At short distances, it has a logarithmic singularity $C_{01} \sim -\ln(r)$, that is, it still diverges at $r = 0$. Since $C_2$ falls off exponentially on the codimension 2 plane inside the D3-brane, the RR charge of the D1-string is screened. This is expected since, due to its coupling to $A_1$, the $C_2$ field is massive; the RR charge from $d\phi \simeq 1/r$ is canceled by that from $A_1 \simeq -1/r$ (see figure 1). On the other hand, the winding number is conserved, so we have a consistent vortex solution of the equation of motion. As we shall discuss in the rest of this paper, this vortex solution represents what a D1-string becomes as it moves inside a D3-brane. For that reason, we refer to this vortex solution as a D1-vortex.

### 3.4 The energy density of a D1-vortex

Let us calculate the tension $\tau$ of the D1-vortex. We are actually more interested in the energy density in the $xy$-plane. The localization of the energy density indicates that the D1-string has not dissolved completely. Varying the action (3.3) with respect to the 00 component of the metric, we get the energy density of the vortex solution,

$$\mathcal{E} = \frac{1}{2}(\partial_r C_{01})^2 + \frac{1}{2}B_z^2 = \frac{a^2 n^2 \xi^2}{2}\left(K_1^2(u) + K_0^2(u)\right) \tag{3.21}$$

where the topological term $C_2 \wedge G_2$ and $C_2 \wedge \delta^2(x_\perp)$ in eq. (3.3) do not contribute since they do not involve the metric. Integrating over the $xy$-plane, we obtain the tension of the vortex

$$\tau = \lim_{r_s \to 0} 2\pi \int_{r_s}^{\infty} \mathcal{E} r dr \simeq -\pi a^2 n^2 \ln(\xi r_s) + \frac{2\pi a^2 n^2}{4}. \tag{3.22}$$

The magnetic field $B_z$ is infinite at $r = 0$ but its contribution to $\tau$ is square integrable, so we have a finite contribution (the second term) to the tension from the magnetic flux (see figure 2). On the other hand, the tension $\tau$ diverges in the limit $r_s \to 0$, due to the



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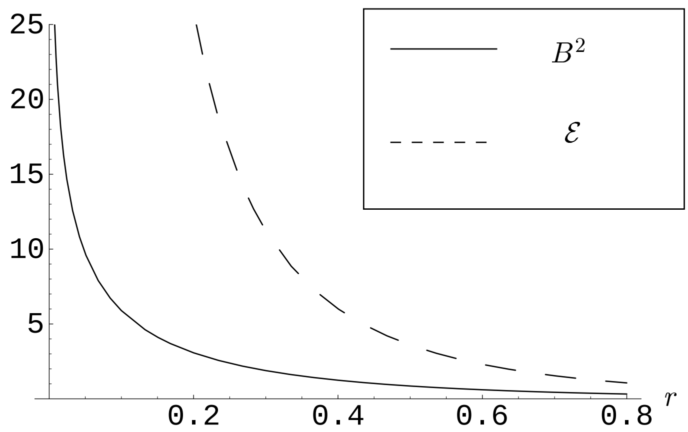


**Figure 2:** The total energy density $\mathcal{E}$ (dashed line) and the energy density in the B field (solid line). Here, $a = n = \xi = 1$.

$(\partial_r C_{01})^2$ term. (Note that the vortex from eq. (3.2) also has a divergent tension.) This divergence is easily interpreted when we compare this system to the abelian Higgs model (see below). We need to think of $r_s$ as the inverse of the scalar Higgs mass, $r_s = 1/m_H$. In this model, this massive Higgs mode is frozen (or ignored), i.e., its mass is effectively set to infinite; this is the origin of the above divergence. This divergence is expected to be removed when we consider a better effective action.

## 4. Discussions

First we show that the above D1-vortex may be viewed as a special limit of the vortex in the abelian Higgs model. Introducing the analogue of the RR charge, we see that a vortex in abelian Higgs model also has a net zero charge. We then summarize the overall physics of the D1-vortex and comment on how the physics may change as we improve the low energy effective theory for this system. We suggest that the resulting system of $n$ D1-strings and a D3-brane is stable as a bound state. Finally, we show that it is straightforward to generalize the stability argument to the case of a D$(p-2)$-brane inside a stack of D$p$-branes.

### 4.1 Relation to the abelian Higgs Model

The simplest way (by far) to see the existence and stability of D1-vortices in a D3-brane is to realize the direct connection between the above vortex solution and a certain limit of the abelian Higgs model. The abelian Higgs model lagrangian is:

$$L = -\overline{D_\mu \Phi} D^\mu \Phi - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\lambda}{4}(\Phi\overline{\Phi} - v^2)^2 \tag{4.1}$$

where $D_\mu = \partial_\mu + ieA_\mu$. After spontaneous symmetry breaking, we have a massive gauge field and a massive Higgs boson:

$$m_A^2 = 2e^2 v^2 \,, \qquad m_H^2 = \lambda v^2 \,. \tag{4.2}$$

The equations of motion are

$$
\begin{aligned}
D^\mu D_\mu \Phi - \frac{\lambda}{2}(\Phi\overline{\Phi} - v^2)\Phi &= 0\,, \\
\partial_\mu G^{\mu\nu} &= 2eIm[\overline{\Phi} D^\nu \Phi] = J^\nu\,.
\end{aligned}
\tag{4.3}
$$

Consider a static vortex along the $z$-axis. In Coulomb gauge, with the ansatz in cylindrical coordinate,

$$
\begin{aligned}
A_\theta &= -\frac{n}{er}\alpha(r)\,, \\
\Phi &= v\beta(r)e^{i\phi}\,, \qquad \phi = n\theta
\end{aligned}
\tag{4.4}
$$

and the equations of motion reduce to

$$
\begin{aligned}
\alpha'' - \frac{\alpha'}{r} - 2e^2v^2\beta^2(\alpha - 1) &= 0\,, \\
\beta'' + \frac{\beta'}{r} - \frac{n^2\beta}{r^2}(\alpha-1)^2 - \frac{\lambda v^2 \beta}{2}(\beta^2 - 1) &= 0\,,
\end{aligned}
\tag{4.5}
$$

where $\alpha(0) = \beta(0) = 0$. At large $r$,

$$
\begin{aligned}
\Phi &= ve^{in\theta}(1 - c_1 e^{-m_H r} + \cdots)\,, \\
A_\theta &= -\frac{n}{er}(1 - c_2 e^{-m_A r} + \cdots)\,,
\end{aligned}
\tag{4.6}
$$

where $c_1$ and $c_2$ are constants that can be determined numerically. These are the well-known Abrikosov-Nielsen-Oleson vortex solutions with winding number $n$ [32, 33].

We may introduce the same 2-form field $C_2$ here. The current $J^\nu$ defined in eq. (3.10) or eq. (4.3) is conserved: $\partial_\nu J^\nu = 0$. In terms of $C_2$,

$$
\epsilon^{\mu\nu\rho\sigma}\partial_\nu C_{\rho\sigma} = \frac{1}{m_A} J^\mu \tag{4.7}
$$

For the vortex solution, we have, for large $r$,

$$
\epsilon^{\theta r 01}\partial_r C_{01} = -\frac{i}{m_A}\Phi^\dagger\left(\frac{1}{r}\partial_\theta \Phi + ieA_\theta\Phi\right) \simeq \frac{n}{m_A r}c_2 e^{-m_A r} + \cdots
$$

so

$$
\oint \epsilon^{\theta r 01}\partial_r C_{01} r d\theta = 0\,.
$$

That is, in terms of the $C_2$ charge, the winding number contribution from $\Phi$ cancels the magnetic flux contribution from $A_\theta$ so that the net $C_2$ charge of the vortex is zero, that is, $C_{01}$ drops off exponentially at large distances from the core. For this reason, the $C_2$ charge is not a useful quantum number to measure the presence/absence of vortices. This is identical to the situation in the D1-vortex case.

Suppose we freeze the Higgs field $|\Phi| = v$ (i.e., $\beta = 1$). Then we are left with only the axionic field $\phi(x)$ (where $\Phi = e^{i\phi}$) and $A_\mu(x)$, and we have only the above equation for $\alpha(r)$, which reduces to exactly eq. (3.12), where $\phi$ and $A_\theta$ are to be identified, with $\xi^2 = m_A^2 = 2e^2v^2$ and $a^2 = 2v^2$. This allows us to identify the D1-vortices as a particular limit of the topologically stable abelian Higgs vortices (crudely speaking, we may achieve this limit by taking $\lambda \to \infty$). The tension of abelian Higgs vortex is

$$\begin{aligned} \tau &= \int r dr d\theta \left[ D_i \Phi D^i \Phi^\dagger + \frac{1}{2} B^2 + V(\Phi) \right], \\ &\simeq 2\pi v^2 \left[ \ln \left( \frac{m_H}{m_A} \right) + 2 + 1 \right], \end{aligned} \tag{4.8}$$

where the first term represents the gradient energy of the scalar field. When we freeze $|\Phi| = v$, we essentially send $m_H \sim 1/r_s \to \infty$, so the D1-vortex tension diverges logarithmically. This logarithmic divergence is what we have obtained earlier. The presence/variation of $|\Phi|$ smooths out the singularity and yields a finite tension. This divergence due to the freezing of $|\Phi|$ is a consequence of the approximation that can be rectified easily.

There remains a somewhat subtle issue to discuss. Normally, the abelian Higgs vortex has $\Phi = 0$ at the center of the core, where the phase (the axion) is not defined and the gauge field is massless. In the above model, $A_1$ is massive everywhere on the brane, so the vortex solution seems to have no 'core'. Actually, the core should be thought as the point where the phase is not defined, even with $|\Phi| = 1$ there. That is, as long as we remove a point from the $xy$-plane, it is no longer simply connected and has non-trivial winding.

## 4.2 The physical picture

We have a consistent vortex solution of the D3-brane action with the bulk contribution coming from the 2-form RR field. One should interpret such a vortex as a D1-string inside a D3-brane (see figure 2), as they have the same winding number. Outside the D3-brane,

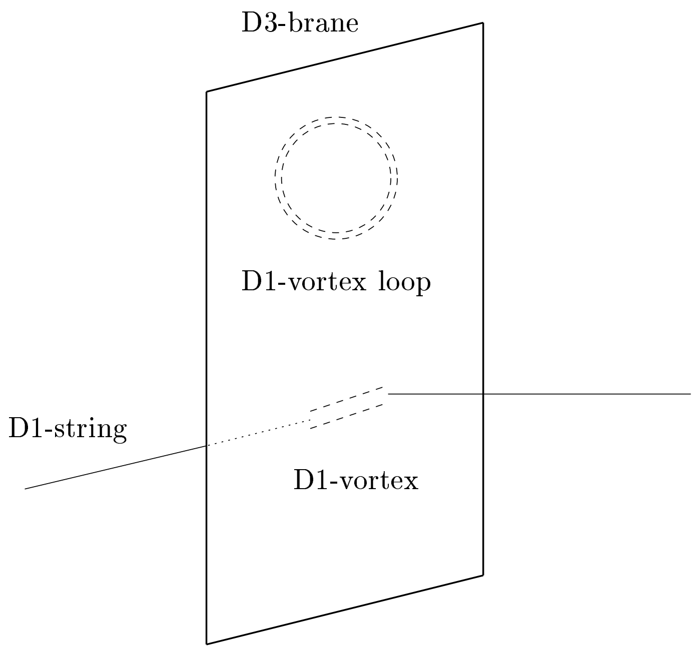


**Figure 3:** A D1-string intersecting a D3-brane and forming a D1-vortex inside the D3-brane. The picture also shows a D1-vortex loop existing on its own.

both $C_2$ and its dual (the axion $\phi$ here) measure the same RR charge of a D1-string, which is identified with the winding number. Inside a D3-brane, they measure different things. $\phi$ again measures the winding number, which is conserved as the D1-string moves inside/outside the D3-brane. On the other hand, $\star dC_2$ measures the combination of $d\phi$ and $A_1$. If one uses $C_2$ to measure the RR charge, one may view this vortex solution as a combination of two different vortices: a positively RR charged vortex ($d\phi \simeq 1/r$) screened by a negative RR charged vortex ($A_1 \simeq -1/r$), resulting in a net zero RR charge as measured by $C_2$. This screening of $C_2$ charge happens because $C_2$ becomes massive. Naively, they look like a D1-string screened by an anti-D1-string. However, they cannot annihilate, since the origin of the RR charge for the D1-component is different from that for the anti-D1-component. Furthermore, the two charges do not cancel locally. As we move this vortex outside a D3-brane, the magnetic flux around the core disappears since the gauge field is absent outside the D3-brane. So this vortex recovers its RR charge and becomes a D1-string. In this sense, we believe the conserved winding number is the appropriate quantity to follow as we move a D1-string inside/outside a D3-brane. In the tachyon condensation picture, a BPS D1-string is a vortex, so here we simply see that the vortex changes its thickness, tension, and magnetic flux as it moves in/out a D3-brane, but it is always topologically stable, due to the winding number measured by the axion $\phi$.

As $\xi \sim g_s^{1/2}$ goes to zero, the vortex grows ($1/\xi$) to infinite size and its energy density spread throughout the D3-brane. The D1-string tension ($\xi^{-2}$) also goes to infinity while the D1-vortex tension grows only logarithmically. So, as a D1-string moves inside a D3-brane, it goes to a lower tension vortex with infinite size, i.e., just like dissolution.

### 4.3 Low energy effective theory in string theory

The $\delta$-function is the origin of the singular behavior of the tension. Including the partner of $C_2$ (i.e., $|\Phi|$) should get rid of the $\delta$-function singularity. In string theory, this massive Higgs mode is always present. Together with $\phi$, they form a complex scalar mode. Consider the low energy effective theory of a generic type-IIB orientifold model. One may write the effective lagrangian for all the string modes as

$$L = L_{0,2} + L_{0,I} + L_M \tag{4.9}$$

where $L_{0,2}$ includes only quadratic terms of the massless modes, while $L_{0,I}$ includes only the interaction terms among the massless modes. All other quadratic and higher dimensional operator terms are included in $L_M$. The action (3.3) is a part of $L_{0,2}$. There is a massive mode $\eta$, with mass $m_\eta$, inside $L_M$. In comparison to the abelian Higgs model, $\eta$ and $\phi$ form the complex field $\Phi = (v + \eta)e^{i\phi}$. All terms involving $\eta$ in the abelian Higgs model (and more) are included in $L_M$. For the ground state, we may ignore $\eta$ or integrate it out. For a vortex solution, since $|\Phi| = 0$ at the core, we must include $\eta$ or $|\Phi|$ in the effective action. As can be seen in the abelian Higgs model, the presence of $\eta$ will smooth out the vortex solution and yield a finite tension $\tau$, i.e., the D1-vortex. To see this feature more closely within superstring theory, one should turn to the tachyon condensation approach, where D1-strings are treated as vortex defects.

One may use boundary superstring field theory (BSFT) to study the tachyon condensation properties. Since the vortex is not BPS inside the D3-brane, its identification with a D1-string remains to be shown. The BSFT approach allows us to identify the same vortex outside a D3-brane as a D1-string (since it has the correct tension and RR charge of a D1-string) and so it is appropriate to consider the corresponding vortex inside a D3-brane as a D1-vortex.

### 4.4 Estimate of the D1-vortex tension

We can make a crude estimate of the D1-vortex tension by supposing that $r_s \approx \xi^{-1}$. In this case, the energy is almost entirely in the $B$ field and we get the tension of the vortex to be:

$$\tau_{\text{vortex}} \simeq \frac{2\pi a^2 n^2}{4} = \pi n^2 \tau_1^2 \kappa_4^2 \,. \tag{4.10}$$

One interesting question is if this solution corresponds to the BPS bound state of a $n$D1-D3 system. If not, then we would expect some kind of instability in the vortex solution. From SUSY arguments, one can obtain the mass of the BPS bound state for $n$ D1-branes and a D3-brane:

$$M_{(nD1-D3)} = \sqrt{n^2\tau_1^2 V_1^2 + \tau_3^2 V_3^2}\,,$$

and the mass of the vortex is therefore expected to be

$$M_{\text{vortex}} = \sqrt{\tau_3^2 V_3^2 + n^2\tau_1^2 V_1^2} - \tau_3 V_3 \approx \frac{n^2\tau_1^2 V_1^2}{2\tau_3 V_3}\,.$$

So the tension is just

$$\tau_{\text{vortex}} \approx \frac{n^2\tau_1^2 V_1}{2\tau_3 V_3} \approx \frac{n^2\tau_1^2}{2\tau_3 V_2}\,.$$

Now, what should we take for $V_2$? Since the vortex is localized, the correct $V_2$ to take would be the size of the vortex in the codimension-2 plane, namely $V_2 \sim \xi^{-2}$. This yields

$$\tau_{\text{vortex}} \approx \frac{n^2\tau_1^2\xi^2}{2\tau_3} \approx n^2\tau_1^2\kappa_4^2$$

which agrees with eq. (4.10) up to an $\mathcal{O}(1)$ factors. Note that the energy of $n$ D1-vortex is proportional to $n^2$, not $n$. In this sense, a D1-vortex is not BPS. However, the above result does suggest that the $n$D1-D3 system saturates the BPS bound and so is a BPS bound state. This is consistent with the D1-vortex stability argument. This means that the solution is not only stable classically due to its topological nature but also quantum mechanically (there should be no stringy topological change).

### 4.5 Generalization to D$(p-2)$-brane inside D$p$-branes

It is fairly easy to generalize this solution to the D$p$-D$(p-2)$ system. Indeed, $\star_{p+1} dC_{p-1} = d\phi$ in $p+1$ dimensions and the number of degree of freedom between $A$ and $C_{p-1}$ match up again. The action for this system is completely similar to what we had before (3.3):

$$S = -\int_{M_{p+1}} \frac{1}{2}|G_2|^2 + \frac{1}{2}|dC_{p-1}|^2 + \xi_p C_{p-1}\wedge G_2 + 2\pi n a \delta^2(x_\perp)\wedge C_{p-1} \tag{4.11}$$

where $\xi_p = \sqrt{2\tau_p}\kappa_{p+1}$.

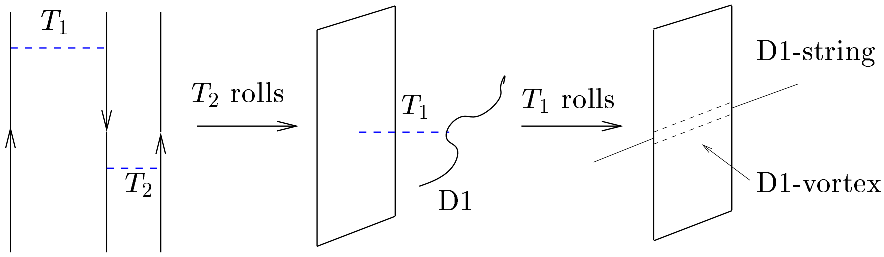


**Figure 4:** The D3-D3-anti-D3 system. The D$\overline{\text{D}}$-branes on which $T_2$ ends are always on top of each other. Creation of a vortex by first forming the D1-string (from the annihilation of the D3-anti-D3 pair) outside the other D3-brane and then bringing it back. Alternatively, one can form the vortex directly on the D3-brane. Arrows up represent D3-branes and the arrow down represents the anti-D3-brane.

One can generalize this result further to a stack of D$p$-branes. In this case, the gauge group is U($N$). Since $C_{p-1}$ couples to the first Chern class,

$$\int_{M_{p+1}} C_{p-1} \wedge \mathrm{Tr}\, F_2 = \int_{M_{p+1}} C_{p-1} \wedge G_2 \tag{4.12}$$

the Green-Schwarz mechanism breaks the U(1) only. Vortex solutions due to this U(1) breaking are straightforward extensions of the above solution.

Based on S-duality, we expect similar properties for F-strings on a D3-brane. It would be interesting to generalize this result to the $(p, q)$-strings.

To summarize, we have found that, contrary to the usual folklore, a D$(p-2)$-brane is stable inside a D$p$-brane and it becomes a D$(p-2)$-vortex. This vortex

- is topologically stable;
- is slightly spread, but still has localized energy density;
- is no longer BPS (when $n$ of them are brought together, their tension increase like $n^2$ instead of $n$);
- becomes a D$(p-2)$-brane when moved outside a D$p$-brane;
- and has net zero RR charge but has a non-zero winding number and quantized magnetic flux.

## 5. D1-string and tachyon condensation

Here we like to study the stability of a D1-string inside a D3-brane from the perspective of tachyon condensation, using the boundary superstring field theory (BSFT) formalism [34]–[37]. We first show its stability by a topological argument, due to the $C_2$ coupling. Then we relate the tachyon condensation approach to the approach discussed earlier. We consider a system of two parallel D3-branes and one anti-D3-brane (see figure 4), where one of the D3-branes is on top of the anti-D3-brane, with the open string tachyon $T_2$ stretching

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between them. When the other D3-brane is separated from this brane-anti-brane pair with separation $\varphi \neq 0$, the annihilation of this pair (described by the rolling of $T_2 \to \infty$) produces defects that are D1-strings outside the remaining D3-brane [38]. If the separation is small, the remnant of $T_1$ becomes a tachyonic mode stretching between a D1-string and the D3-brane. When the three branes are all on top of each other ($\varphi = 0$), any defects produced are inside the remaining D3-brane. This model allows us to study the stability of a D1-string inside a D3-brane, or whether it can be formed. This approach also reveals the connection between the D1-string and the D1-vortex.

To set up the problem, let us keep track of all scalar modes that may trigger spontaneous symmetry breaking of the relevant maximum gauge symmetry $\mathrm{U}(2) \times \mathrm{U}(1)$. First, there is the moduli $\varphi$ that measures the separation between the two D3-branes. For $\varphi \neq 0$, $\mathrm{U}(2) \to \mathrm{U}(1)_1 \times \mathrm{U}(1)_2$. There is also the tachyon mode $T$ which is a SU(2) doublet and there is the bulk RR field $C_2$, which couples to a particular combination of U(1)s. The degeneracy structure of the final vacuum tells us the existence of any defect.

### 5.1 A topological argument

Let us start by considering the setup where all three branes are on top of each other (see figure 4). The gauge group of this system is $\mathrm{U}(2) \times \mathrm{U}(1) = \mathrm{SU}(2) \times \mathrm{U}(1)_{DD} \times \mathrm{U}(1)_{\overline{D}} = \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_L$ where the gauge field assignments are (see table 1 for the various definitions of U(1)s)

$$\mathrm{U}(1)_{DD} \to B_1\,, \qquad \mathrm{U}(1)_{\overline{D}} \to B_2\,, \qquad \mathrm{U}(1)_Y \to B = \frac{(B_1 - \sqrt{2}B_2)}{\sqrt{3}}\,.$$

The tachyon $T$ couples only to $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$:

$$D_\mu T = \left(\partial_\mu + \frac{ig}{2}\begin{pmatrix} A_3 + B_1 - \sqrt{2}B_2 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -A_3 + B_1 - \sqrt{2}B_2 \end{pmatrix}\right)\begin{pmatrix} T_1 \\ T_2 \end{pmatrix}. \tag{5.1}$$

The relative normalization (i.e., the $\sqrt{2}$ factor) is fixed by moving either of the D3-brane away from the pair [35, 36].

| | |
|---|---|
| $\mathrm{U}(1)_{DD}$ | $B_1$ |
| $\mathrm{U}(1)_{\overline{D}}$ | $B_2$ |
| $\mathrm{U}(1)_1$ | $A_1 = \frac{1}{\sqrt{2}}(A^3 + B_1)$ |
| $\mathrm{U}(1)_2$ | $A_2 = \frac{1}{\sqrt{2}}(-A^3 + B_1)$ |
| $\mathrm{U}(1)_Y$ | $B = (B_1 - \sqrt{2}B_2)/\sqrt{3}$ |
| $\mathrm{U}(1)_\gamma$ | $A = \frac{1}{2}(A^3 + \sqrt{3}B)$ |
| $\mathrm{U}(1)'$ | $Z = \frac{1}{2}(-A^3 + \sqrt{3}B)$ |
| $\mathrm{U}(1)_C$ | $\frac{1}{\sqrt{2}}(A_2 + B_2)$ |

**Table 1:** U(1)s and their gauge fields.

First, let us ignore the $C_2$ field. Without loss of generality, consider the tachyon rolling:

$$T = \begin{pmatrix} 0 \\ T_2 \end{pmatrix} . \tag{5.2}$$

In BSFT, $T_2 \to \infty$, though a canonical field redefinition brings the *vev* to a finite value. So $T_2$ breaks $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$ to $\mathrm{U}(1)_\gamma$ with gauge field $A$. This is just like the spontaneous symmetry breaking in the standard electroweak model. So, in the absence of $C_2$, it has trivial homotopy and no vortex is possible:

$$\begin{aligned} \mathrm{SU}(2) \times \mathrm{U}(1)_Y &\overset{T_2 \neq 0}{\to} \mathrm{U}(1)_\gamma \,, \\ \Pi_1 \left( \frac{\mathrm{SU}(2) \times \mathrm{U}(1)_Y}{\mathrm{U}(1)_\gamma} \right) &= \mathbb{1} \,. \end{aligned} \tag{5.3}$$

Suppose we naively try to construct a vortex choosing $T_2 \sim e^{i\theta}$. One may identify this vortex as a D1-string inside a D3-brane. However, this vortex is unstable due to the rolling of $T_1$. One may view $T_1$ (actually its remnant) as the tachyonic D1-D3 open string mode. Topologically, the degenerate vacuum is $S^3$, so any $S^1$ on $S^3$ will shrink and disappear.

Now, let us introduce $C_2$ and its relevant couplings. The various field strengths are

$$\begin{aligned} \mathcal{A}^1 &= \begin{pmatrix} (A_3 + B_1)/\sqrt{2} & W^+ \\ W^- & (-A_3 + B_1)/\sqrt{2} \end{pmatrix} , \\ F^1 &= d\mathcal{A}^1 + \mathcal{A}^1 \wedge \mathcal{A}^1 \,, \\ F^2 &= dB_2 \,, \\ F^- &= F^1 - F^2 = d\mathcal{A}^1 + \mathcal{A}^1 \wedge \mathcal{A}^1 - dB_2 \,. \end{aligned}$$

The relevant coupling terms (between $\mathrm{C}_2$ and the gauge field strengths) in the RR action for the $\mathrm{DD\overline{D}}$ system are [39, 40]

$$S_{RR} \propto \int_{M_4} C_2 \wedge \left( \mathrm{Tr}(F^-) + \frac{1}{t}(e^{-\lambda t} - 1)(\mathrm{Tr}(\mathcal{T} F^1 - t F^2) + \cdots) \right) \tag{5.4}$$

where $\lambda = 2\pi\alpha'$ and

$$\begin{aligned} t &= T^\dagger T = T_1 \overline{T}_1 + T_2 \overline{T}_2 \,, \\ \mathcal{T} &= T T^\dagger = \begin{pmatrix} T_1 \overline{T}_1 & T_1 \overline{T}_2 \\ T_2 \overline{T}_1 & T_2 \overline{T}_2 \end{pmatrix} . \end{aligned}$$

For $T = 0$, the above $C_2$ coupling reduces to

$$\int_{M_4} C_2 \wedge d(B_1 - \sqrt{2} B_2) \propto \int_{M_4} C_2 \wedge dB \tag{5.5}$$

So, as discussed earlier, the $\mathrm{U}(1)_Y$ is spontaneously broken by this coupling, with

$$\Pi_1 \left( \mathrm{U}(1)_Y \right) |_{C_2} = \mathbb{Z} \,. \tag{5.6}$$

The interesting case is when $T_2$ is non-zero. For such a *vev*, tachyon condensation will break the gauge group down to $\mathrm{U}(1)_\gamma$, as explained earlier. After tachyon condensation, the $\mathrm{D}\overline{\mathrm{D}}$ pair together with their gauge fields disappear. As $T_2 \to \infty$, $Z$ becomes massive and decouples, while the other $\mathrm{U}(1)_C$ (with gauge field $(A_2 + B_2)/\sqrt{2}$) of the $\mathrm{D}\overline{\mathrm{D}}$ pair is expected to be confined [41, 42]. (The confinement of $\mathrm{U}(1)_C$ can give rise to electric flux vortices.) The only gauge field remaining after this tachyon condensation is the $\mathrm{U}(1)_1$ on the remaining D3-brane. Putting $T_1 = 0$ but not $T_2$ in eq. (5.4) and dropping the $W^\pm$ fields which play no role here:

$$
\begin{aligned}
S_{RR} \propto & \int_{M_4} C_2 \wedge \left(\frac{1}{\sqrt{2}} d(A^3 + B_1)\right) + C_2 \wedge \left(e^{-\lambda T_2 \overline{T}_2} d(\frac{1}{\sqrt{2}}(-A^3 + B_1) - 2B_2)\right) \\
\to & \int_{M_4} C_2 \wedge dA_1 \, . \qquad (5.7)
\end{aligned}
$$

As $T_2 \to \infty$, $C_2$ couples only to the remaining $\mathrm{U}(1)_1$. So $\mathrm{U}(1)_1$ is broken by its coupling to $C_2$. The fundamental homotopy group is now

$$\Pi_1 \left(\mathrm{U}(2) \times \mathrm{U}(1)\right) = \mathbb{Z} \, , \qquad (5.8)$$

or, more specifically,

$$\Pi_1 \left(\mathrm{U}(1)_1\right) |_{C_2} = \mathbb{Z} \, . \qquad (5.9)$$

We conclude that vortex solutions due to the $C_2$ coupling are topologically allowed and stable, even though the particular U(1) that is spontaneously broken by $C_2$ changes as the tachyon rolls. To summarize, of the 3 U(1)s, namely $\mathrm{U}(1)' \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_C$, $\mathrm{U}(1)'$ is Higgsed by $T_2$, $\mathrm{U}(1)_1$ is Higgsed by $C_2$ and $\mathrm{U}(1)_C$ is confined. There is no destabilizing tachyonic mode left so a vortex (due to the $C_2$ breaking of an abelian gauge symmetry) is topologically stable as was discussed in section 3. This is the so-called D1-vortex, and the effective action (3.3) is the simplest action that captures this key property.

To obtain a D1-string outside a D3-brane, we turn on $\varphi \neq 0$. The gauge symmetry is $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \times \mathrm{U}(1)_{\overline{D}}$. For large enough $\varphi$, $T_1$ is no longer tachyonic (it is a normal massive mode). The $D\overline{D}$ pair annihilation yields D1-strings as vortices in the bulk, following from the $T_2$-rolling and

$$\Pi_1 \left(\mathrm{U}(1)'\right) |_{T_2} = \mathbb{Z} \qquad (5.10)$$

where the gauge field of $\mathrm{U}(1)'$ is $Z = (-A^3 + \sqrt{3}B)/\sqrt{2} = (A_2 - B_2)/\sqrt{2}$. The identification of these BPS vortices as D1-strings is justified as they have the correct tension and RR charge (see below). As $\varphi$ decreases to around the string scale, $T_1$ becomes tachyonic and starts to roll, signaling the instability of a D1-string. Magnetic flux on the D3-brane gathers to screen the RR charge of the D1-string. As $\varphi \to 0$, the D1-string becomes a D1-vortex. Note that $A_1$ is always present as the key component of the abelian Higgs vortex. Although the actual dynamics can be a little complicated, we expect the transition from a D1-string (a $\mathrm{U}(1)'$ vortex due to tachyon condensation) to a D1-vortex (a $\mathrm{U}(1)_1$ vortex due to $C_2$ coupling) to be smooth.

### 5.2 The DD$\overline{\mathrm{D}}$ system in boundary superstring field theory

In principle, one can solve the D1-string to D1-vortex transition using BSFT. In practice, this is quite complicated. Here, as a modest step, let us draw a closer contact between the BSFT approach and the effective action (3.3) approach. First, consider the effective action for the D$p$-anti-D$p$-brane pair in boundary superstring field theory [37], where the tachyon is a complex field (not be confused with the tachyon doublet of the DD$\overline{\mathrm{D}}$ system).

$$S_{(Dp\overline{D}p)} = -\tau_p \int d^{p+1}x\sqrt{-g}\, 2e^{-\lambda T\overline{T}}\, \mathcal{F}(\mathcal{X}+\sqrt{\mathcal{Y}})\, \mathcal{F}(\mathcal{X}-\sqrt{\mathcal{Y}}) \tag{5.11}$$

where

$$\mathcal{X} \equiv 2\pi\alpha'^2 g^{\mu\nu}\partial_\mu T\partial_\nu \overline{T}\,, \qquad \mathcal{Y} \equiv \left(2\pi\alpha'^2\right)^2 \left(g^{\mu\nu}\partial_\mu T\partial_\nu T\right)\left(g^{\alpha\beta}\partial_\alpha \overline{T}\partial_\beta \overline{T}\right),$$

and the function $\mathcal{F}(x)$ is given by [34]

$$\mathcal{F}(x) = \frac{4^x x\Gamma(x)^2}{2\Gamma(2x)} = \frac{\sqrt{\pi}\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)}\,, \tag{5.12}$$

$$\mathcal{F}(x) = \begin{cases} 1+(2\ln 2)x+\mathcal{O}(x^2)\,, & 0<x\ll 1\,, \\ \sqrt{\pi x}\,, & x\gg 1\,. \end{cases} \tag{5.13}$$

Annihilation of the brane pair happens when $T\to\infty$. Tachyon condensation also allows the creation of codimension-2 defects. For

$$T = uz^n = u(x+iy)^n\,, \qquad u\to\infty \tag{5.14}$$

we have a vortex at the origin in the $xy$-plane, whose tension and RR charge are [35, 36, 37]

$$\tau = 4\pi^2 n\alpha'\tau_p = n\tau_{p-2}\,, \qquad \mu_{RR} = n\mu_{p-2} = n\tau_{p-2}g_s \tag{5.15}$$

which are precisely the properties of $n$ BPS D$(p-2)$-branes, allowing us to identify these defects as D$(p-2)$-branes. These "vortices" are BPS with respect to each other, that is, the total tension of $n$ parallel static vortices (with total RR charge $n\mu_{p-2}$) is $n\tau_{p-2}$, independent of their relative positions.

Next, let us consider the action for the DD$\overline{\mathrm{D}}$ system. This action cannot be written in closed form because of the mixing terms between the two complex tachyons [40]. However, for the tachyon doublet profile (large $u$)

$$T = \begin{pmatrix} 0 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ uz \end{pmatrix}, \tag{5.16}$$

the mixing terms are unimportant and the action reduces to something very similar to eq. (5.11)

$$S_{(Dp\overline{D}p)} = -\tau_p \int d^{p+1}x\sqrt{-g}\,(1+2e^{-\lambda T_2\overline{T}_2}\, \mathcal{F}(\mathcal{X}+\sqrt{\mathcal{Y}})\, \mathcal{F}(\mathcal{X}-\sqrt{\mathcal{Y}}))\,. \tag{5.17}$$

Now, we need to restore the gauge field $A_1$ (under which $T$ is neutral) on the brane. The simplest way would be to add the DBI factor such that when $T_2 = 0$, we get back the DBI action on the remaining D3-brane. This gives (keeping only terms to the order we need)

$$\begin{aligned} S_{\mathrm{DD}\overline{\mathrm{D}}} = & -\tau_3 \int d^4x \left( \sqrt{-|g+\lambda G_2|} + 2\sqrt{-g}e^{-\lambda T_2 \overline{T}_2}\mathcal{F}^2 \right) - \frac{1}{4\kappa_4^2}|dC_2|^2 - \\ & -\mu_3 \int_{M_4} \left( C_2 \wedge G_2 + i\lambda^2 e^{-\lambda T_2 \overline{T}_2} C_2 \wedge dT_2 \wedge d\overline{T}_2 \right) \end{aligned} \tag{5.18}$$

where the arguments of $\mathcal{F}$ are given in eq. (5.17) and we have added the relevant terms of the RR action [39, 40]. The last term is the source term for $C_2$. A tachyon profile of $T_2 = uz$ with $u \to \infty$ gives us a $\delta$-function for this term and this is the D1-string solution outside the D3-brane. Inside the brane, a modified $T_2$ may smooth this $\delta$-function.

The equations of motion for this action (expanding the DBI part) are

$$\begin{aligned} \frac{1}{4\kappa_4^2} d \star dC_2 &= \mu_3 G_2 + i\lambda^2 \mu_3 e^{-\lambda T\overline{T}} dT \wedge d\overline{T}\,, \\ \tau_3 \lambda^2 d \star dA_1 &= \mu_3 dC_2\,. \end{aligned} \tag{5.19}$$

There is also an equation of motion for $T$ but it is somewhat complicated. Again, if we use the naive profile $T = uz$ with $u \to \infty$, we get back the equations of motion in section 3

$$\begin{aligned} i\mu_3 \lambda^2 e^{-\lambda T\overline{T}} dT \wedge d\overline{T} &= \frac{\mu_1 \lambda u^2}{2\pi} 2e^{-\lambda u^2(x^2+y^2)} dx \wedge dy \\ &\stackrel{u\to\infty}{\to} \mu_1 \delta^2(x,y) dx \wedge dy\,. \end{aligned} \tag{5.20}$$

With a different profile for $T$, the solution for both $A_1$ and $C_2$ will change, but the qualitative feature of a vortex is clear. It would be interesting to solve this system to obtain an explicit tachyon profile.

## 6. Remarks and conclusion

In this paper we have shown that there exists a consistent vortex solution of the D3-brane action with RR 2-form bulk terms, provided the $C_2$ coupling is non-zero. This requires the compactification of the extra dimensions, as is the case in any brane world scenario. This vortex solution is not BPS, but it has a localized energy density, not spread throughout the D3-brane. As we pull this vortex out of a D3-brane, it becomes a BPS D1-string; that is, the D1-string is never broken, although its properties do change as it moves inside/outside a D3-brane. The transition from a D1-string to a D1-vortex may be followed in tachyon condensation.

In terms of tachyon condensation, all D$p$-branes (maybe with the exception for $p = 9$) may be viewed as solitonic defects. So there are only odd $p$-defects in type-IIB theory. If a D1-string is a boundary of a domain wall, or if a D1-string breaks so that it has ends, then such domain walls or string ends will appear as even dimensional defects (2-dim. and 0-dim. respectively), which presumably should not exist in type-IIB theory. The resolution proposed in this paper is consistent with this belief.



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It will be interesting to study the interaction and intercommutation properties of these D1-vortices. In cosmology, these defects may become cosmic strings in our universe. We need to determine their properties to find any distinct signature that would differentiate them from the usual quantum field theory vortices. The importance of this study cannot be overstated since this is, at the moment, a most promising window into superstring theory and the inflationary scenario.

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